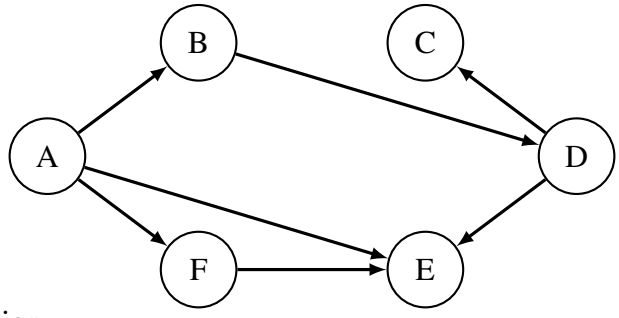
CS 61B Summer 2020 | Small Group Tutoring Section 10: Graphs | Worksheet 10



1 Graph Representation

Represent the graph above with an adjacency list and an adjacency matrix representation.

Solution: Depending on the convention being used, nodes may or may not have edges to themselves. For this problem, we stick with the convention that nodes do **not** have an edge to themselves.

Adjacency List

A → [B, E, F]
B → [D]
C → []
D → [C, E]
E → []
F → [E]

FROM / TO

	A	B	C	D	E	F
A	0	1	0	0	1	1
B	0	0	0	1	0	0
C	0	0	0	0	0	0
D	0	0	1	0	1	0
E	0	0	0	0	0	0
F	0	0	0	0	1	0

(in the above matrix 0 means false and 1 means true)

Note: Edge lists and adjacency lists are **not** the same! An edge list is a list stored in each node that contains all successors and possibly predecessors of that node (see lecture). An adjacency list is more of a table (or map data structure) that lists the adjacent vertices for each vertex in the graph; it is a representation of the graph as a whole. Graphs are commonly represented using adjacency lists and matrices but be aware of what is meant by an edge list.

2 Searches and Traversals

Run depth first search (DFS) preorder and breadth first search (BFS) on the graph above, starting from node A. List the order in which each node is first visited. Whenever there is a choice of which node to visit next, visit nodes in alphabetical order. **Solution:** Preorder means we visit the node *before* visiting its children.

DFS preorder: A, B, D, C, E, F
BFS: A, B, E, F, D, C

3 Topological Sorting

Give a valid topological ordering of the graph. Is the topological ordering of the graph unique?

Solution: A topological ordering is a linear ordering of nodes such that for every directed edge S $\rightarrow$ T, S is listed before T. For this problem, the topological ordering of the graph is **not** unique. Below, we list two valid topological orderings for the graph.

- One valid ordering: A, B, D, C, F, E
  - Explanation: One way to approach this problem is to take any node with no edges leading to it and return it as the next node. After returning a node, we delete it and any edges leaving from it and look for a node with no incoming edges in the updated graph. We can repeat this until we have no nodes left. If at any point in this process we have a multiple choices for which node to return then the topological ordering is not unique.
- Another possible valid ordering: A, F, B, D, E, C
  - Explanation: Note that this ordering is just the reverse of DFS postorder traversal. Reverse DFS postorder will always be a valid topological ordering. This is because a DFS postorder traversal visits nodes only after all successors have been visited, so the reverse traversal visits nodes only after all predecessors have been visited.

Note: Only Directed Acyclic Graphs (DAGs), which are directed graphs that do not contain any cycles, have topological orderings. This is because within any given cycle, no one node comes before another. There are no valid topological orderings for undirected graphs because there is no direction associated with any edge. No one node comes before another, so it does not make sense to have a topological ordering for undirected graphs.

4 Dijkstra's Algorithm

(a) Given the following graph, run Dijkstra's algorithm starting at node *A*. For each iteration, write down the entire state of the algorithm. This includes the value `dist(v)` for all vertices *v*. Keep track of the vertices traversed along the shortest paths from A to every other node in the graph. You will need to maintain an `edgeTo` array.

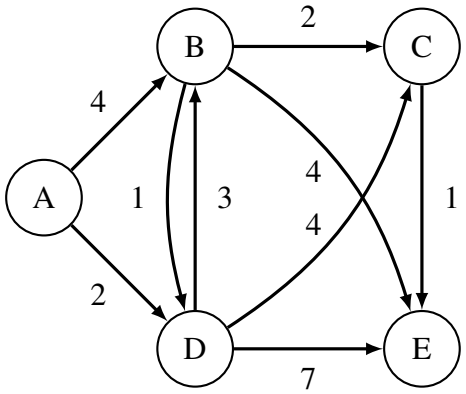
Solution: To run Dijkstra's algorithm, start with `dist(v)` for all vertices *v* set to ∞ and a fringe that includes all the vertices. The fringe is a minimum priority queue that orders the vertices by `dist(v)` values.

At each iteration, pop off a node from the fringe (this will be the vertex in the fringe with the lowest `dist(v)`). For each outgoing edge *e* from this popped vertex, check to see whether the sum of `dist(`*popped*`)` and the edge *e*'s value is less than the current `dist` value of the vertex the edge connects to. If so, set the `dist` value of that vertex to this lower value.

Note that vertices that have already been popped from the fringe will never have their `dist` values changed. This is because when we pop off a vertex, the distance to that vertex can only increase by considering other vertices and their edges (since the popped vertex currently has the minimum `dist` value).

Continue until all nodes have been popped from the fringe.

	`dist`(v)					
v	Init	Pop A	Pop D	Pop B	Pop C	Pop E
A	**0**	0	0	0	0	0
B	∞	4	4	**4**	4	4
C	∞	∞	6	6	**6**	6
D	∞	2	**2**	2	2	2
E	∞	∞	9	8	7	**7**

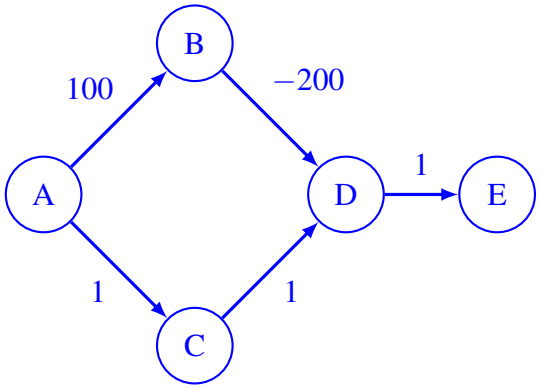


Solution: Here is the final edgeTo array that we get after running Dijkstra's algorithm to completion. `edgeTo`(A) is "-" because it was used as the starting vertex. `edgeTo`(C) is D because it is the first `edgeTo` vertex that was encountered along the shortest path to C. We would get D for `edgeTo`(C) if we update the edgeTo and dist arrays when a candidate path is strictly shorter than the existing path. We would get B for `edgeTo`(C) if we update the edgeTo and dist arrays when a candidate path is shorter than or the same distance as the existing path.

v	`edgeTo`(v)
A	-
B	A
C	D
D	A
E	C

(b) What must be true about our graph in order to guarantee Dijkstra's will return the shortest path's tree to every vertex? Draw an example of a graph that demonstrates why Dijkstra's might fail if we do not satisfy this condition.

Solution: In order to guarantee Dijkstra's will return the shortest path to every vertex, we must have a graph that has no negative edge weights. Take the following graph as an example of why negative edge weights might cause an error:



For this graph, if we ran Dijkstra's starting from A, then we would get the incorrect shortest path to E since we would choose the bottom path through C instead of the top path through B.

We choose the bottom path because we reach and pop off vertices C, D, and E before popping off vertex B and considering its edge to D. This is because in Dijkstra's, when we pop off a vertex, we do so with the assumption that the distance to that vertex can only increase by considering other vertices and their edges (since the popped vertex currently has the min dist value). With negative edges, this assumption is no longer true.

Note that having negative edge weights does not guarantee Dijkstra's will fail, but if we have all non-negative edge weights then we are guaranteed to get the shortest path. When working with distances from the real world, we don't have to worry about negative edge weights because all distances in reality are strictly non-negative.